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In the following, thermometer Y was in sun, with its bulb coated with lamp-black in gold size:—

		r_o	r_a	p	r_v	$\frac{1}{r_v}$	App. Alt.	cosec.
1861.	h m	°	°	°	°		° '	
Aug. 6	8 7 A.M.	16·2	19·1	6·6	25·7	·0389	31 17	1·926
	9 8	18·0	21·2	7·7	28·9	·0346	42 21	1·484
	9 32	18·3	21·6	8·0	29·6	·0338	45 12	1·409

Note.—Referring to the method of computing the Sun's potential temperature, described in the proceedings of the Society for March 1860, and employing the same rule with R, the extra atmospheric value of r equal to 70° at Earth's mean distance, we arrive at 12,880,000° as the potential temperature of its radiating surface.*

If we expose the flame of a bat's-wing-jet to one ball of a differential thermometer, the effect is the same, whether the broad side or the narrow side of the flame is presented, as I have found on trial. Now the potential temperature being equal to the product of r by the reciprocal of the angular space occupied by the flame, it is in the one case about five times greater than in the other. In the same way we might compute the potential temperature of an angular space occupied by many thousand flames placed one behind the other, extending in a line from the observer, and probably we should find it cumulative in the ratio of the number of flames.

From observations I have made on gas flames with the radiation meter, fig. 1, it would seem to require about 4000 bat's-wing-flames ranged behind each other to give a potential equal to that of the Sun.

If the upper radiating matter of the Sun is in any degree transparent or permeable to radiation from lower strata, it is obvious that the *actual* temperature may thus be much below the *potential.*

26 *Royal Crescent, Edinburgh, November* 25, 1861.

Note on a Theorem of Jacobi's, in relation to the Problem of Three Bodies. By A. Cayley, Esq.

The following theorem of Jacobi's (*Comptes Rendus*, t. iii., p. 61 (1836)) has not, I think, found its way in an explicit form into any treatise of physical astronomy. The theorem is as follows, viz. "Consider the movement of a point without mass round the Sun, disturbed by a planet the orbit of which

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* There is a typographical error in the *Monthly Notices*, page 201, line 32; for $x = 918000$, read $x = 9{,}180{,}000$.

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is circular. Let $x\,y\,z$ be the rectangular co-ordinates of the disturbed body, the orbit of the disturbing planet being taken as the plane of $x\,y$, and the Sun as the centre of co-ordinates; let a' be the distance of the disturbing planet, $n'\,t$ its longitude, m' its mass, M the mass of the Sun: then we have, *rigorously*,

$$\frac{1}{2}\left[\left(\frac{d\,x}{d\,t}\right)^2+\left(\frac{d\,y}{d\,t}\right)^2+\left(\frac{d\,r}{d\,t}\right)^2\right]-n'\left(x\,\frac{d\,y}{d\,t}-y\,\frac{d\,x}{d\,t}\right)$$

$$=\frac{\mathrm{M}}{(x^2+y^2+z^2)^{\frac{1}{2}}}+m'\left\{\frac{1}{(x^2+y^2+z^2-2\,a'\,(x\cos n'\,t+y\sin n'\,t)+a'^2)^{\frac{1}{2}}}\right.$$

$$\left.-\frac{x\cos n'\,t+y\sin n'\,t}{a'^2}\right\}+\mathrm{C}.$$

This is therefore a new integral equation, which, in the problem of three bodies, subsists, as regards the terms independent of the excentricity of the disturbing planet, and which is rigorous as regards all the powers of the mass of such planet. In the *Lunar Theory* the Earth must be substituted in the place of the Sun, and the Sun taken as the disturbing planet."

To prove the theorem, as expressed in polar co-ordinates, I take the equations of motion in the form in which I have employed them in my "Memoir on the Theory of Disturbed Elliptic Motion" (*Memoirs*, vol. xxvii. p. 1 (1859)), viz.

$$\frac{d}{d\,t}\,\frac{d\,r}{d\,t}-r\cos^2 y\left(\frac{d\,v}{d\,t}\right)^2-r\left(\frac{d\,y}{d\,t}\right)^2+\frac{n^2a^3}{r^2}=\frac{d\,\Omega}{d\,r},$$

$$\frac{d}{d\,t}\left(r^2\cos^2 y\,\frac{d\,v}{d\,t}\right)=\frac{d\,\Omega}{d\,v},$$

$$\frac{d}{d\,t}\left(r^2\,\frac{d\,y}{d\,t}\right)+r^2\cos y\sin y=\frac{d\,\Omega}{d\,y},$$

where

$$\Omega=m'\left\{\frac{1}{\sqrt{r^2+r'^2-2\,r\,r'\cos\mathrm{H}}}-\frac{r\cos\mathrm{H}}{r'^2}\right\};$$

or, since $\cos\mathrm{H}=\cos y\cos(v-v')$, and the Sun is considered as moving in a circular orbit (i.e. $r'=a'$, $v'=n'\,t$), we have

$$\Omega=m'\left\{\frac{1}{\sqrt{r^2+a'^2-2\,r\,a'\cos y\cos(v-n'\,t)}}-\frac{r\cos y\cos(v-n'\,t)}{a'^2}\right\};$$

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so that Ω is a function of r, v, y and of t, which last quantity enters only in the combination $v - n't$. Hence the complete differential coefficient of Ω is

$$\frac{d\,(\Omega)}{dt} = \frac{d'\,\Omega}{dt} - n'\frac{d\,\Omega}{dv},$$

where $\frac{d'\,\Omega}{dt}$ denotes, as usual, the differential coefficient in regard to the time, in so far as it enters through the co-ordinates r, v, y of the disturbed body.

We have, as usual,

$$\frac{d}{dt}\,\frac{dr^2 + r^2\,(\cos^2 y\, dv^2 + dy^2)}{dt^2} = \frac{d'\,\Omega}{dt};$$

but from the foregoing equation,

$$\frac{d'\,\Omega}{dt} = \frac{d\,(\Omega)}{dt} + n'\frac{d\,\Omega}{dv}$$

$$= \frac{d\,(\Omega)}{dt} + n'\frac{d}{dt}\left(r^2\cos^2 y\,\frac{dv}{dt}\right);$$

and, substituting this value, transposing, and integrating, we have

$$\left(\frac{dr}{dt}\right)^2 + r^2\left\{\cos^2 y\left(\frac{dv}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2\right\} - n'\,r^2\cos^2 y\,\frac{dv}{dt} = \Omega + \mathrm{C},$$

which is Jacobi's equation expressed in terms of the co-ordinates r, v, y.

M. de Pontécoulant, in his *Lunar Theory* (1846), where the solar excentricity is neglected, writes (p. 91),

$$\int d'\mathrm{R} = \mathrm{R} + m\int\frac{d\mathrm{R}}{dv}\,dt,$$

($n' = m\,n = m$, since n is there put equal to unity); and combining this with the equation (p. 43),

$$\frac{r^2\,dv}{(1+s^2)\,dt} = h + \int\frac{d\mathrm{R}}{dv}\,dt,$$

we have

$$\int d'\mathrm{R} = \mathrm{R} - m\,h + \frac{m\,r^2\,dv}{(1+s^2)\,dt};$$

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and substituting this value of $\int d' \mathrm{R}$ in the integral of *Vis Viva* (p. 41),

$$\left(\frac{d r}{d t}\right)^2 + \frac{r^2 d v^2}{(1+s^2) d t^2} + \frac{r^2 d s^2}{(1+s^2)^2 d t^2} - \frac{2}{r} + \frac{1}{a} = 2\int d' \mathrm{R},$$

we have what is, in fact, Jacobi's equation.

A Third Memoir on the Problem of Disturbed Elliptic Motion. By A. Cayley, Esq. (Abstract.)

The object of the Memoir is to obtain the differential equations for determining

r, the radius vector,
v, the longitude,
y, the latitude,

of the disturbed body, when the last two co-ordinates are measured in respect to an arbitrarily varying plane (which, however, to fix the ideas, is called the variable ecliptic) and departure-point, or origin of longitudes therein. This is very readily effected by means of an expression for the *Vis-Viva* function given in my "Supplementary Memoir on the Problem of Disturbed Elliptic Motion," *Mem. Roy. Ast. Soc.*, vol. xxviii. pp. 217–234 (1859).

Neglecting the squares of the variations of the variable ecliptic, and also the products of the variations by sin y or $\frac{d y}{d t}$, then (as might be expected) it is found that the equations for r and v are the same as for a fixed ecliptic, and the equation for y is found in a simple form, which is ultimately reduced to coincide with that obtained for the lunar theory by Laplace in the seventh book of the *Mécanique Céleste*, and which is used by him to show that the effect of the variation of the ecliptic on the latitude of the Moon (as measured from the variable ecliptic) is insensible. And it is shown conversely how the approximate formula of the Memoir may be obtained by a process similar to that made use of in the *Mécanique Céleste.*